A whopper of a number

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Models and theories

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- Models:
  - $(\mathbb{Z}, +, 0)$
  - $(\mathbb{R}^{\times}, \times, 1)$.

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Theorem (Gödel's compactness theorem)

A theory T is satisfiable if and only if every finite subset of T is satisfiable.

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Ask Dave. □

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  1. $1 < \omega$
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- $\omega =$ big.
- Exists thing that looks like $\mathbb{N}$, but has a *really* big number in it.